

# Statistics

## Lecture 5



Feb 19-8:47 AM

### Class QZ 3

Consider the Sample  
below

20	23	18	25
30	25	19	24
32	28		

68% Range

$\bar{x} \pm s$   
 $= 24 \pm 5 \rightarrow [19 \text{ to } 29]$

95% Range

$\bar{x} \pm 2s$   
 $= 24 \pm 2(5) \rightarrow [14 \text{ to } 34]$

Find

$$1) \bar{x} = 24.4 \approx 24 \quad \left. \begin{array}{l} \text{Round} \\ \text{to} \\ \text{whole#} \end{array} \right\}$$

$$2) s = 4.648 \approx 5 \quad \left. \begin{array}{l} \text{Round} \\ \text{to} \\ \text{whole#} \end{array} \right\}$$

$$3) s^2 = \frac{108}{5} \quad \left. \begin{array}{l} \text{Reduced} \\ \text{Fraction} \end{array} \right\}$$

Find Z-Score for  
data element 30.

$$Z = \frac{x - \bar{x}}{s} = \frac{30 - 24}{5} = \frac{6}{5}$$

$$-2 \leq Z \leq 2 = 1.2$$

usual element

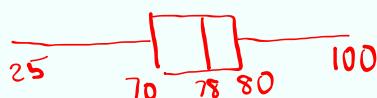
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Class QZ 4

Consider the 5-Number Summary below

Min	25	Q <sub>1</sub>	70	Med	78	Q <sub>3</sub>	80	Max	100
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1) Draw box plot



2) Find IQR

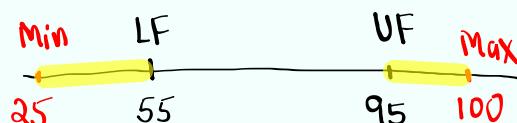
$$IQR = Q_3 - Q_1 = 10$$

3) Find upper fence

$$\begin{aligned} UF &= Q_3 + 1.5(IQR) \\ &= 80 + 1.5(10) = 95 \end{aligned}$$

4) Find lower fence

$$\begin{aligned} LF &= Q_1 - 1.5(IQR) \\ &= 70 - 1.5(10) \\ &= 55 \end{aligned}$$



Outliers 25 to 55 or 95 to 100

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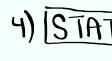
find  $\bar{x}$ ,  $s$ , and  $s^2$  for the group data

below.

1) find class MP

class limits	class F	class MP
18 - 30	5	24
31 - 43	8	37
44 - 56	10	50
57 - 69	7	63

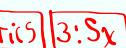
2) class MP  $\rightarrow$  L13) class F  $\rightarrow$  L2

4)   List L1  
  

$$\bar{x} = 45.23$$

for  $s^2$ 

$$S = 13.434$$

$$n = 30$$

$$S^2 = \frac{157001}{870}$$

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Below data are Scores for 25 randomly

Selected exams.

$$1) n=25$$

58 63 65 69

70 72 74 75

75 78 80 82

82 82 85 86

88 90 93 95

96 97 100 100

105

Find  $P_{15}$

$$L = \frac{15}{100} \cdot 25 = 3.75 \rightarrow L=4$$

2) STEM Plot

5 | 8

6 | 359

7 | 024558

8 | 0222568

9 | 03567

10 | 005

$$P_{15} = 4^{\text{th}} \text{ element} = \boxed{69}$$

15% 85%

69

Find  $P_{50}$  (Median)

$$L = \frac{50}{100} \cdot 25 = 12.5 \rightarrow L=13$$

$P_{50} = 13^{\text{th}}$  element

= 82

50% 50%

Find  $P_{80}$

$$L = \frac{80}{100} \cdot 25 = 20$$

$P_{80} = \frac{20^{\text{th}} \text{ element} + \text{Next}}{2}$

=  $\frac{95 + 96}{2} = \boxed{95.5}$

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5 | 8  
6 | 359  
7 | 024558  
8 | 0222568  
9 | 03567  
10 | 005

find  $K$  Such that

$$P_K = 75 \rightarrow \text{Below}$$

$$K = \frac{B}{n} \cdot 100, \text{ whole \%}$$

$$= \frac{7}{25} \cdot 100 = 28$$

$$\boxed{P_{28} = 75}$$

Find  $K$  Such that

$$P_K = 80 \rightarrow \text{Below}$$

$$K = \frac{B}{n} \cdot 100, \text{ whole \%}$$

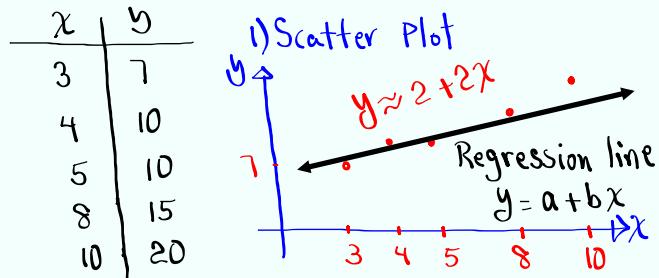
$$= \frac{10}{25} \cdot 100 = 40$$

$$\boxed{P_{40} = 80}$$

$$\boxed{\text{SG 5-8}} \checkmark$$

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Consider the chart below



clear all lists

$x \rightarrow L1$ ,  $y \rightarrow L2$

**STAT**  $\rightarrow$  **CALC**

**2:2-Var Stats**

$$\sum x = 30$$

$$\sum x^2 = 214$$

$$n = 5$$

$$\sum y = 62$$

$$\sum y^2 = 874$$

$$\sum xy = 431$$

**STAT**  $\rightarrow$  **CALC**

**8:LinReg(a+bx)**

$$a = 1.988 \approx 2$$

$$b = 1.735 \approx 2$$

$$r^2 = 0.973 \approx 97\%$$

$$r = 0.987$$

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How to find  $a$  &  $b$ :

$$a = \frac{\sum y \sum x^2 - \sum x \sum xy}{n \sum x^2 - (\sum x)^2} = \frac{62 \cdot 214 - 30 \cdot 431}{5 \cdot 214 - 30^2} = \frac{338}{170} \approx 1.988$$

$$\sum x = 30$$

$$\sum y = 62$$

$$\sum x^2 = 214$$

$$\sum y^2 = 874$$

$$n = 5$$

$$\sum xy = 431$$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{5 \cdot 431 - 30 \cdot 62}{5 \cdot 214 - 30^2} = \frac{295}{170} = 1.735$$

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$r^2$  Coef. of determination

Always in whole %.

It tells us what % of  $y$ -values are explained by  $x$ -values

From Last example  $r^2 \approx 97\%$ .

97% of  $y$ -values are explained by  $x$ -values, 3% are unexplained.

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$r$  Linear Correlation Coef.

$$-1 \leq r \leq 1$$

When  $r$  is close to  $\pm 1$ ,

Linear Correlation is Significant.

When  $r$  is close to 0,

Linear Correlation is not Significant.

From Last example  $r = .987$

Very close to 1  $\rightarrow$  Significant.

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How to compute  $r$ :

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$\begin{aligned} \sum x &= 30 & \sum y &= 62 & = \frac{5 \cdot 431 - 30 \cdot 62}{\sqrt{5 \cdot 214 - 30^2} \sqrt{5 \cdot 874 - 62^2}} \\ \sum x^2 &= 214 & \sum y^2 &= 874 & = \frac{295}{\sqrt{170} \sqrt{526}} \\ n &= 5 & \sum xy &= 431 & = \frac{295}{\sqrt{89420}} = 0.987 \end{aligned}$$

295 [ $\circ$ ] [2nd] [ $x^2$ ] 89420 [enter]

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Making Predictions:

If  $r$  is significant  $\Rightarrow$  use regression line

If  $r$  is not significant  $\Rightarrow$  use  $\bar{y}$

using last example, Predict  $y$  when  $x=6$

1) Assume  $r$  is significant

$$y \approx 2 + 2x = 2 + 2(6) \approx 14$$

2) Assume  $r$  is not significant.

$$\text{use } \bar{y} = \frac{\sum y}{n} = \frac{62}{5} = 12.4$$

SG 9 ✓

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SG 10-13  
Introduction to probabilities  
chances for an event  
to happen.

$E \rightarrow$  Desired event

$P(E) \rightarrow$  Prob. that  $E$  happens.

$$P(E) = \frac{\text{Total # of only desired outcomes}}{\text{Total # of all outcomes}}$$

12 Students

8 Females

Select one student

$$P(\text{Select a female}) = \frac{8}{12} = \boxed{\frac{2}{3}}$$

A standard deck of playing cards  
have 52 cards, 26 Red, 4 aces.

$$P(\text{Select a red card}) = \frac{26}{52} = \boxed{\frac{1}{2}} = 0.5$$

$$P(\text{Select an ace}) = \frac{4}{52} = \boxed{\frac{1}{13}}$$

4  $\div$  52 Math Frac Enter

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Acceptable answers:

1) Reduced fraction

2) Decimal but rounded to  
3-dec. places  
when needed

3) Scientific Notation.

A standard deck of playing cards  
has 52 cards, 12 face cards, 4 Aces.

Draw one card,

$$P(\text{Draw Face or Ace}) = \frac{12+4}{52} = \frac{16}{52} = \boxed{\frac{4}{13}} = 0.308$$

$$P(\text{Draw Face and Ace}) = \frac{0}{52} = \boxed{0}$$

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## Some rules and terminology

- 1)  $0 \leq P(E) \leq 1$
- 2) Sum of all probabilities is 1.
- 3)  $P(E) = 1 \iff$  Sure event
- 4)  $P(E) = 0 \iff$  Impossible event
- 5)  $0 < P(E) \leq .05 \iff$  Rare event

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If we randomly select one person,  
find the prob. that he/she has  
a birthday

1) today  $\frac{1}{365} \approx .003$   
Rare event

2) this month  $\frac{1}{12} = .083$   
Not a rare event

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$E \rightarrow$  Desired event

$P(E) \rightarrow$  Prob. that  $E$  happens

$\bar{E} \rightarrow E\text{-bar, } E\text{-complement, not } E$

$$P(E) + P(\bar{E}) = 1$$

$$P(\bar{E}) = 1 - P(E)$$

### Complement Rule

Suppose  $P(E) = .004$

1) Write  $P(E)$  in % notation

$$.004 = \boxed{.4\%}$$

2) Write  $P(E)$  in reduced fraction

$$.004 \text{ [Math]} \text{ [I: } \text{ Frac } \text{ [Enter]} \quad \frac{1}{250}$$

3) Find  $P(\bar{E})$

$$P(\bar{E}) = 1 - P(E)$$

$$= 1 - .004 = \boxed{.996}$$

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Select one number from

1 2 3 4 ... 36 37 38 39 40

$$1) P(\text{select 5}) = \frac{1}{40}$$

$$2) P(\text{select below 5}) = \frac{4}{40} = \frac{1}{10}$$

$$3) P(\text{select 35 or above}) = \frac{6}{40} = \frac{3}{20}$$

$$4) P(\text{select } \boxed{\text{below 5}} \text{ or } \boxed{\text{at least 35}}) \\ = \frac{4+6}{40} = \frac{10}{40} = \boxed{\frac{1}{4}}$$

$$5) P(\text{select below 5 and at least 35})$$

$$= \frac{0}{40} = \boxed{0}$$

Impossible event.

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I surveyed 100 voters.  
 chart below shows outcome.  
 Question was Do You Support ICE operation?

	Yes	No	Total
Democrat	15	35	50
Republican	40	10	50
Total	55	45	100

Select one of them

$$1) P(\text{Yes}) = \frac{55}{100} = 0.55 \quad 2) P(\text{Democrat}) = \frac{50}{100} = 0.5$$

$$3) P(\text{Yes and Democrat}) = \frac{15}{100} = 0.15$$

$$4) P(\text{Yes or Democrat}) = \frac{90}{100} = 0.9$$

SG 10 ✓

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Addition Rule:

Keyword OR

one action event

$$P(A \text{ or } B) = P(A) + P(B) - P(\text{Both})$$

$$P(A) = 0.7, P(B) = 0.6, P(A \text{ and } B) = 0.4$$

$$1) P(\bar{A}) = 1 - P(A) = 1 - 0.7 = 0.3$$

$$2) P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

↑  
 addition  
 Rule

$$= 0.7 + 0.6 - 0.4 = 0.9$$

$$3) P(\bar{A \text{ or } B}) = 1 - P(A \text{ or } B)$$

$$= 1 - 0.9 = 0.1$$

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$$P(HB) = .35$$

$$P(FF) = .45$$

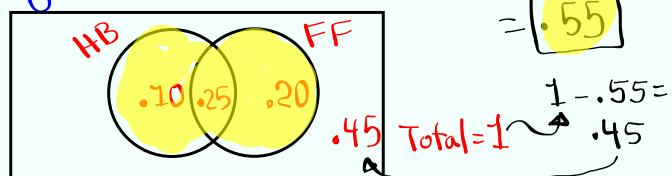
$$P(HB \text{ and } FF) = .25$$

$$1) P(\overline{HB}) = 1 - .35 = .65$$

$$2) P(FF) = 1 - .45 = .55$$

$$3) P(HB \text{ or } FF) = P(HB) + P(FF) - P(HB \text{ and } FF)$$

$$\text{Using Venn Diagram} = .35 + .45 - .25$$

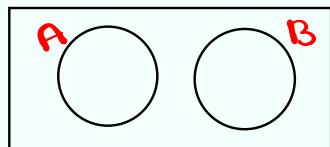


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Mutually Exclusive Events

Disjointed events

$$P(A \text{ and } B) = 0$$



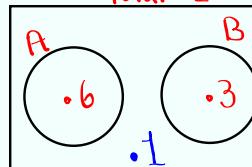
$$P(A) = .6 \quad P(B) = .3$$

$A \notin B$  are  
M.E.E.  
Total = 1

$$1) P(\overline{A}) = 1 - P(A) = .4$$

$$2) P(\overline{B}) = 1 - P(B) = .7$$

$$3) P(A \text{ and } B) = 0$$



$$4) P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .6 + .3 - 0 = .9$$

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class QZ 5

$x$	$y$
2	5
3	9
5	12
6	12
8	15

use  $\text{LinReg}(a+bx)$  to  
find

$$1) a = 3.316 \approx 3$$

$$2) b = 1.518 \approx 2$$

$$3) r^2 = .918 \approx 92\% \quad \left. \right\} \text{whole \%}$$

$$4) r = .958 \quad \left. \right\} \text{3-dec.}$$

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