

# Statistics

## Lecture 5



Feb 19-8:47 AM

Class QZ 3

Consider the Sample  
below

20	23	18	25
30	25	19	24
32	28		

68% Range

$$\bar{x} \pm S$$

$$= 24 \pm 5 \rightarrow \boxed{19 \text{ to } 29}$$

95% Range

$$\bar{x} \pm 2S$$

$$= 24 \pm 2(5) \rightarrow \boxed{14 \text{ to } 34}$$

Find

$$1) \bar{x} = 24.4 \approx \boxed{24}$$

$$2) S = 4.648 \approx \boxed{5}$$

$$3) S^2 = \boxed{\frac{108}{5}}$$

} Round  
to  
whole #

} Reduced  
Fraction

Find Z-Score for  
data element 30.

$$Z = \frac{x - \bar{x}}{S} = \frac{30 - 24}{5} = \frac{6}{5}$$

$$= \boxed{1.2}$$

$$-2 \leq Z \leq 2$$

usual element

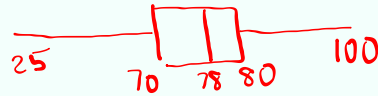
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Class QZ 4

Consider the 5-Number Summary below

Min  $Q_1$  Med  $Q_3$  Max  
 25 70 78 80 100

1) Draw box Plot



2) Find IQR

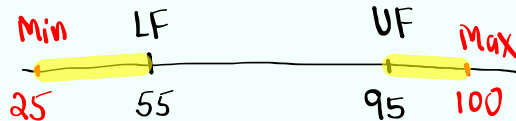
$$IQR = Q_3 - Q_1 = 10$$

3) Find upper Fence

$$UF = Q_3 + 1.5(IQR) \\ = 80 + 1.5(10) = 95$$

4) Find lower Fence

$$LF = Q_1 - 1.5(IQR) \\ = 70 - 1.5(10) \\ = 55$$



outliers 25 to 55 or 95 to 100

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find  $\bar{x}$ ,  $s$ , and  $s^2$  for the group data below.

1) find class MP

class limits	class F	class MP
18 - 30	5	24
31 - 43	8	37
44 - 56	10	50
57 - 69	7	63

2) class MP  $\rightarrow$  L13) class F  $\rightarrow$  L24) [STAT]  $\rightarrow$  Calc

1:1-Var Stats

List L1

Freq List L2

[Calculate]

$$\bar{x} = 45.23$$

$$s = 13.434$$

$$n = 30$$

for  $s^2$ 

[VARS] [5: Statistics] [3: Sx]

[ $x^2$ ] [Math] [1:  $\rightarrow$  Frac] [Enter]

$$s^2 = \frac{157001}{870}$$

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Below data are Scores for 25 randomly  
Selected exams.

58 63 65 69  
70 72 74 75  
75 78 80 82  
82 82 85 86  
88 90 93 95  
96 97 100 100  
105

$$1) n = 25$$

2) STEM Plot

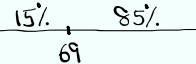
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5 | 8
6 | 359
7 | 024558
8 | 0222568
9 | 03567
10 | 005
  
```

Find  $P_{15}$

$$P_{15} = 4\text{th element} = \boxed{69}$$

$$L = \frac{15}{100} \cdot 25 = 3.75 \rightarrow L = 4$$

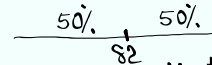


Find  $P_{50}$  (Median)

$$P_{50} = 13\text{th element}$$

$$L = \frac{50}{100} \cdot 25 = 12.5 \rightarrow L = 13$$

$$= 82$$



Find  $P_{80}$

$$P_{80} = \frac{20\text{th element} + \text{Next one}}{2}$$

$$L = \frac{80}{100} \cdot 25 = 20$$

$$= \frac{95 + 96}{2} = \boxed{95.5}$$

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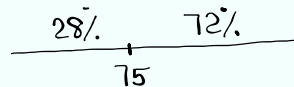
5 | 8
6 | 359
7 | 024558
8 | 0222568
9 | 03567
10 | 005
  
```

find  $K$  such that

$$P_K = 75$$

$$K = \frac{B}{n} \cdot 100, \text{ whole \%}$$

$$= \frac{7}{25} \cdot 100 = 28$$

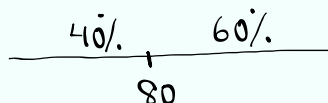


$$\boxed{P_{28} = 75}$$

find  $K$  such that  $P_K = 80$

$$K = \frac{B}{n} \cdot 100, \text{ whole \%}$$

$$= \frac{10}{25} \cdot 100 = 40$$



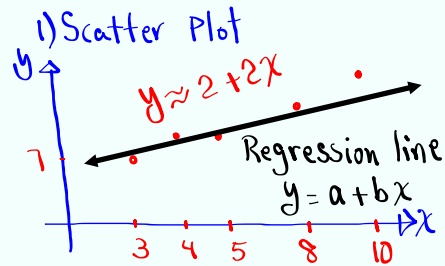
$$P_{40} = 80$$

$\boxed{\text{SG } 5-8} \checkmark$

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Consider the chart below

$x$	$y$
3	7
4	10
5	10
8	15
10	20



Clear all lists

$x \rightarrow L1$ ,  $y \rightarrow L2$

[STAT] → CALC

2: 2-Var Stats

$$\sum x = 30$$

$$\sum y = 62$$

$$\sum x^2 = 214$$

$$\sum y^2 = 874$$

$$n = 5$$

$$\sum xy = 431$$

[STAT] → CALC

8: LinReg(a+bx)

$$a = 1.988 \approx 2$$

$$b = 1.735 \approx 2$$

$$r^2 = .973 \approx 97\%$$

$$r = .987$$

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How to find a & b:

$$a = \frac{\sum y \sum x^2 - \sum x \sum xy}{n \sum x^2 - (\sum x)^2} = \frac{62 \cdot 214 - 30 \cdot 431}{5 \cdot 214 - 30^2} = \frac{338}{170} \approx \boxed{1.988}$$

$$\sum x = 30$$

$$\sum y = 62$$

$$\sum x^2 = 214$$

$$\sum y^2 = 874$$

$$n = 5$$

$$\sum xy = 431$$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{5 \cdot 431 - 30 \cdot 62}{5 \cdot 214 - 30^2} = \frac{295}{170} = \boxed{1.735}$$

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$r^2$  Coef. of determination  
Always in whole%

It tells us what % of  $y$ -values  
are explained by  $x$ -values

From Last example  $r^2 \approx 97\%$

97% of  $y$ -values are explained  
by  $x$ -values, 3% are unexplained.

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$r$  Linear Correlation Coef.

$$-1 \leq r \leq 1$$

When  $r$  is close to  $\pm 1$ ,

Linear Correlation is  
Significant.

When  $r$  is close to 0,

Linear Correlation is not  
Significant.

From Last example  $r = .987$

Very close to 1  $\rightarrow$  Significant.

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How to compute r:

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$\begin{aligned} \sum x &= 30 & \sum y &= 62 & & = \frac{5 \cdot 431 - 30 \cdot 62}{\sqrt{5 \cdot 214 - 30^2} \sqrt{5 \cdot 874 - 62^2}} \\ \sum x^2 &= 214 & \sum y^2 &= 874 & & \\ n &= 5 & \sum xy &= 431 & & = \frac{295}{\sqrt{170} \sqrt{526}} \\ & & & & & = \frac{295}{\sqrt{89420}} = \boxed{.987} \\ & & & & & 295 \div \boxed{2nd} \boxed{x^2} 89420 \boxed{enter} \end{aligned}$$

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Making Predictions:

If r is Significant  $\Rightarrow$  Use regression line

If r is not Significant  $\Rightarrow$  Use  $\bar{y}$

using last example, Predict y when  $x=6$

1) Assume r is Significant

$$y \approx 2 + 2x = 2 + 2(6) \approx \boxed{14}$$

2) Assume r is not Significant.

$$\text{use } \bar{y} = \frac{\sum y}{n} = \frac{62}{5} = \boxed{12.4}$$

SG 9 ✓

Jan 14-5:41 PM

Introduction to probabilities Sig 10-13  
 chances for an event  
 to happen.  
 $E \rightarrow$  Desired event  
 $P(E) \rightarrow$  Prob. that  $E$  happens.  

$$P(E) = \frac{\text{Total \# of only desired outcomes}}{\text{Total \# of all outcomes}}$$

12 Students

8 Females

Select one student

$$P(\text{Select a female}) = \frac{8}{12} = \boxed{\frac{2}{3}}$$

A standard deck of playing cards  
 have 52 cards, 26 Red, 4 aces.

$$P(\text{Select a red Card}) = \frac{26}{52} = \boxed{\frac{1}{2}} = \boxed{.5}$$

$$P(\text{Select an ace}) = \frac{4}{52} = \boxed{\frac{1}{13}}$$

4  $\div$  52 Math 1:  $\rightarrow$  frac Enter

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Acceptable answers:

1) Reduced Fraction

2) Decimal but rounded to  
 3-dec. places  
 when needed

3) Scientific Notation.

A standard deck of playing cards  
 has 52 cards, 12 face cards, 4 Aces.  
 Draw one card,

$$P(\text{Draw Face or Ace}) = \frac{12+4}{52} = \frac{16}{52} = \boxed{\frac{4}{13}} = \boxed{.308}$$

$$P(\text{Draw Face and Ace}) = \frac{0}{52} = \boxed{0}$$

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Some rules and terminology

1)  $0 \leq P(E) \leq 1$

2) Sum of all probabilities is 1.

3)  $P(E) = 1 \iff \text{Sure event}$

4)  $P(E) = 0 \iff \text{Impossible event}$

5)  $0 < P(E) \leq .05 \iff \text{Rare event}$

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If we randomly select one person,  
find the prob. that he/she has  
a birthday

1) today  $\frac{1}{365} \approx .003$   
Rare event

2) this month  $\frac{1}{12} = .083$   
not a rare event

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$E \rightarrow$  Desired event

$P(E) \rightarrow$  Prob. that  $E$  happens

$\bar{E} \rightarrow$  E-bar, E-Complement, not  $E$

$$P(E) + P(\bar{E}) = 1$$

$$P(\bar{E}) = 1 - P(E)$$

### Complement Rule

Suppose  $P(E) = .004$

1) write  $P(E)$  in % notation

$$.004 = .4\%$$

2) write  $P(E)$  in reduced fraction

$$.004 \text{ (Math) (1:} \rightarrow \text{Frac) (Enter) } \frac{1}{250}$$

3) find  $P(\bar{E})$

$$P(\bar{E}) = 1 - P(E)$$

$$= 1 - .004 = .996$$

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Select one number from

1 2 3 4 . . . 36 37 38 39 40

$$1) P(\text{select } 5) = \frac{1}{40}$$

$$2) P(\text{select below } 5) = \frac{4}{40} = \frac{1}{10}$$

$$3) P(\text{select } 35 \text{ or above}) = \frac{6}{40} = \frac{3}{20}$$

$$4) P(\text{select below } 5 \text{ or at least } 35) \\ = \frac{4+6}{40} = \frac{10}{40} = \frac{1}{4}$$

$$5) P(\text{select below } 5 \text{ and at least } 35) \\ = \frac{0}{40} = 0$$

Impossible event.

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I surveyed 100 Voters.  
 chart below shows outcome.  
 Question was Do You Support ICE  
 operation?

	Yes	NO	Total
Democrat	15	35	50
Republican	40	10	50
Total	55	45	100

Select one of them

$$1) P(\text{Yes}) = \frac{55}{100} = .55 \quad 2) P(\text{Democrat}) = \frac{50}{100} = .5$$

$$3) P(\text{Yes and Democrat}) = \frac{15}{100} = .15$$

$$4) P(\text{Yes or Democrat}) = \frac{90}{100} = .9$$

SG 10

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Addition Rule:

Keyword OR  
 one action event

$$P(A \text{ or } B) = P(A) + P(B) - P(\text{A and B})$$

Both

$$P(A) = .7, P(B) = .6, P(A \text{ and } B) = .4$$

$$1) P(\bar{A}) = 1 - P(A) = 1 - .7 = .3$$

$$2) P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

↑  
addition  
Rule

$$= .7 + .6 - .4 = .9$$

$$3) P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B)$$

$$= 1 - .9 = .1$$

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$$P(HB) = .35$$

$$P(FF) = .45$$

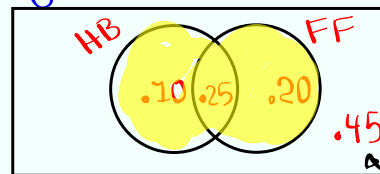
$$P(HB \text{ and } FF) = .25$$

$$1) P(\overline{HB}) = 1 - .35 = \boxed{.65}$$

$$2) P(\overline{FF}) = 1 - .45 = \boxed{.55}$$

$$3) P(HB \text{ or } FF) = P(HB) + P(FF) - P(HB \text{ and } FF)$$

Using Venn Diagram



$$= .35 + .45 - .25 = \boxed{.55}$$

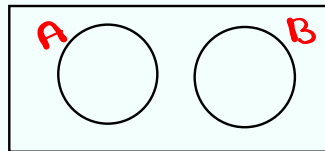
$$Total = 1 \quad \begin{matrix} 1 - .55 = \\ .45 \end{matrix}$$

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Mutually Exclusive Events

Disjoint events

$$P(A \text{ and } B) = 0$$



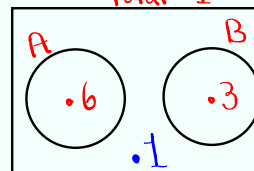
$$P(A) = .6 \quad P(B) = .3$$

$$1) P(\overline{A}) = 1 - P(A) = \boxed{.4}$$

A & B are  
M.E.E.  
Total = 1

$$2) P(\overline{B}) = 1 - P(B) = \boxed{.7}$$

$$3) P(A \text{ and } B) = \boxed{0}$$



$$4) P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .6 + .3 - 0 = \boxed{.9}$$

Jan 14-7:02 PM

Class Q7 5

$x$	$y$
2	5
3	9
5	12
6	12
8	15

Use  $\text{LinReg}(a+bx)$  to  
Find

$$1) a = 3.316 \approx 3$$

$$2) b = 1.518 \approx 2$$

$$3) r^2 = .918 \approx 92\%$$

$$4) r = .958$$

} Round  
to  
whole

} whole  
%

} 3-dec.

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